# Microscopic Simulation of Pedestrian Crowd Motion

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We present an investigation of a driven random-walk model which is used to simulate pedestrian motion. We analyse the dependency of the global density and the mean velocity on the boundary density for open and periodic boundary conditions. Global and local fundamental diagrams are presented for a random sequential and parallel update. In addition, an analytical solution for the relation between mean velocity and density is presented, too.

## 1 Introduction

The simulation of traffic and pedestrian motion has gained much attention in recent years [8,15]. The increasing size of recreation facilities makes it necessary to investigate the motion of pedestrians and different properties of human motion. For example it is useful to know how to lead persons through a shopping mall or how to increase the efficiency of an evacuation process onboard a cruise ship [16,17]. The latter point is interesting from the view of ship designers to create corridors and stairs which are able to provide the possibility for a fast evacuation on the one hand and to provide as many cabins as possible on the other hand. It is also interesting for insurance companies to evaluate the risk for passengers on such a ship.

For modelling of pedestrian motion different types of simulations are used. Macroscopic models [9] that focus on the optimisation of pedestrian flows or different types of microscopic models: continuos in space [5] utilising a so called social-force model and models which are discrete in space. The latter are called cellular automaton models if they are also discrete in time and the state variable of the cell [3]. They are as well concerned with fundamental model characteristics [2,11,12,13,19] and their application [10].

There is, however, a considerable lack of empirical data and only a limited number of studies have been performed [4,6,7]. For an overview see [5,14,20,21]. Especially with regard to single-person data like walking speed the data basis is scarce. However, there are investigations under way as shown in other articles of these proceedings.

The outline of this article is as follows. Next we describe our model. After that we present results from simulations with a random sequential and parallel update under open and periodic boundary conditions. The last but one chapter presents an analytical approach for the relation between the mean velocity and the density. The final chapter gives an outlook to outstanding tasks.

## 1.1 Model

In 1999 Muramatsu et. al. [1] presented a driven random walk model for pedestrian motion. The pedestrians move on a square lattice. The size of the cells is supposed to be 40 cm. The pedestrians are allowed to move one cell per time step.

The decision for the new position of a pedestrian is made as follows. First, the pedestrian counts the neighbouring empty cells. From that number a probability for each unoccupied cell is computed and by means of a random number the new position is chosen. The formulas to calculate the probabilities are listed in table 1. A backstep is not allowed. In case of an occupied cell the probability for that direction is set to zero.

| **Direction** | **Probability** |
|---|---|
| *forward* | $D + \frac{1-D}{\#\,\text{unoccupied cells}}$ |
| *up* | $\frac{1-D}{\#\,\text{unoccupied cells}}$ |
| *down* | $\frac{1-D}{\#\,\text{unoccupied cells}}$ |

**Table 1:** Probabilities for the three possible walking directions.

Figure 1 shows the layout of the corridor. The arrows mark the possible moves of the pedestrians. The amount of possible moves depends on the number of unoccupied cells around each pedestrian.

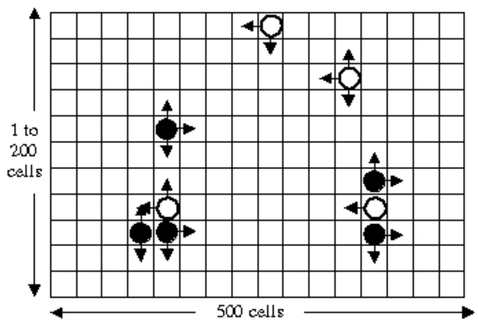


**Figure 1:** Layout of the corridor and possible moves of the pedestrians.

Under open boundary conditions the pedestrians were added at the left or right boundary, depending on the walking direction. The density at the boundaries was fixed at simulation startup.

The pedestrians were not able to cross the upper or lower boundary.

### 1.2 Update Procedures

We used two different update procedures for the simulations. The random sequential update (rsu) moves the pedestrians in a random order but every pedestrian only once in a timestep. After the move of a pedestrian the next pedestrian reacts on the new distribution. Because of this there is no problem with multiple occupations of a single cell.

The parallel update moves the pedestrians in the first step only virtually. Due to this there is the possibility for a multiple occupation of single cells because the moving pedestrian does not know if the cell he has chosen is still empty or occupied by a virtually moved pedestrian. Because of this after the virtual move of all pedestrians a conflict solution is performed to resolve the problems with more than one pedestrian per cell. A cell which is occupied by more than one pedestrian is drawn under the concerned pedestrians. The winner gets the cell, the losers move back to their old positions.

## 2 Simulation Results

The simulations were carried out with the parameters mentioned in table 2. For a width of $w = 100$ cells the fraction between the two walking directions was additionally set to $c = 1/3$ and $c = 1/5$ to study the dependency of the density for the transition from moving to jammed pedestrians from the fraction.

| **Width** | **Drift** | **Fraction** | **Simulated timesteps** | **Update** | **Boundary conditions** |
|---|---|---|---|---|---|
| *10, 20, 50, 200* | *0.0, 0.1, 0.4, 0.7* | *0.5* | *10000* | *rsu* | *open* |
| *100* | *0.0, 0.1, 0.4, 0.7* | *0.2, 0.3, 0.5* | *10000* | *rsu* | *open* |
| *1, 2, 5, 10* | *0.0, 0.25, 1.0* | *0.0, 0.2, 0.5* | *50000* | *rsu, parallel* | *open, periodic* |

**Table 2:** Parameters for the simulations.

For the corridor with widths smaller than 10 cells a drift $D = 0.25$ was chosen because in the case of three unoccupied cells in the neighbourhood this value means a probability of $p_{\text{forward}} = 0.5$ for a step in forward direction.

Figure 2 shows the mean velocity versus the density at the boundaries for open boundary conditions simulated with the rsu. There is only a slight dependency of the transition from moving to stopped pedestrians on the drift. A similar picture is recorded for periodic boundary conditions. With the parallel update the transition to the stopped phase occurs at a lower density due to the conflict solution. An increasing drift leads to an increasing mean velocity in the moving phase, up to a value of 1 for drift $D = 1$.

Additionally we recorded global and local fundamental diagrams. The local fundamental diagrams recorded the flow and the density on each lane for each walking direction. The global fundamental diagrams just distinguish between the walking directions and not the different lanes.

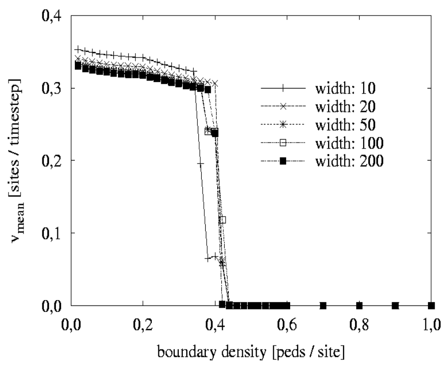


**Figure 2:** Mean velocity versus boundary density for rsu with open boundaries.

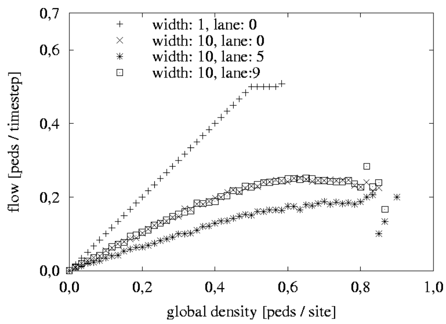


**Figure 3:** Local fundamental diagrams for the rsu with open boundary conditions.

We found three different local fundamental diagrams. Fig. 3 shows the local fundamental diagrams for corridors with a width of $w = 1$ cell and $w = 10$ cells. The fundamental diagram for the corridor with width $w = 1$ cell is independent from the drift due to the fact that the drift increases the probability for a forward step. Because there is no possibility for another direction than forward in a corridor with a width of $w = 1$ cell, the probability for the forward step is always $p_{\text{forward}} = 1$ and so not depending on the drift.

The flow on lanes which are at the upper or lower boundary is higher than the flow on inner lanes due to a missing third direction on a boundary lane. The fact that on boundary lanes the maximum number of unoccupied cells is two increases the probability for these two directions. On inner lanes the maximum number of unoccupied cells is three. This means that in case of a drift $D = 0$ the probability for all three directions is $p = 1/3$. For the same case on a boundary lane the probability for each of the two directions is $p = 1/2$. This results in a higher flow on the boundary lanes.

As a last result we present a graph (Fig. 4) of simulated timesteps until all pedestrians are jammed. We altered the program to simulate either 1.000.000 timesteps or stop the simulation when the mean velocity reached the value 0. We performed 30 recursions for each density for the rsu with periodic boundaries. It was found that for high densities the amount of simulated timesteps was in a range of some hundred timesteps. For medium densities the number of timesteps increased due to an increasing amount of possible configurations.

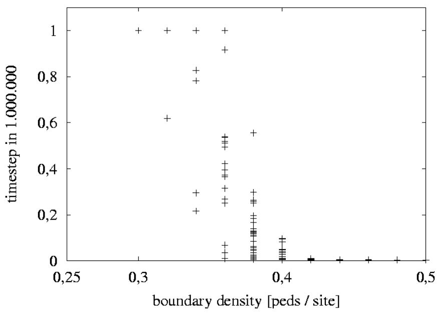


**Figure 4:** Simulated timesteps until a mean velocity of 0 occurs.

For a density of 0.3 there was no jam found in the system. All 1.000.000 timesteps were performed with still a non-zero mean velocity.

# 3 Analytical Solution

We tried to formulate an analytical solution for the speed-density relation. The idea is that the mean velocity equals the probability of a certain configuration to arise multiplied with the probability to choose the forward step.

There are three different configurations which provide the possibility for a forward step.

An empty cell occurs with a probability of $(1-\rho)$ and an occupied cell occurs with a probability of $\rho$.

1. The first configuration is the one with all three neighbouring cells unoccupied. This configuration arises with a probability of $p_{\text{conf.}} = (1-\rho)^3$.

   The probability for the forward step in this case is $p_{\text{forward}} = D + \frac{1-D}{3}$.

2. In the second configuration one of the upper or lower neighbouring cells is occupied. The probability for that configuration is $p_{\text{conf.}} = (1-\rho)^2 \cdot \rho$.

   A forward step is made with a probability of $p_{\text{forward}} = D + \frac{1-D}{2}$.

   This case occurs in two different forms: either the upper neighbouring cell is occupied or the lower neighbouring cell. Due to that, the probability has to be multiplied with a factor 2.

3. The last case is the one with only the front cell unoccupied. The forward step has a probability of $p_{\text{forward}} = 1$ and this configuration arises with a probability of $p_{\text{conf.}} = (1-\rho) \cdot \rho^2$.

   Writing this all together leads to the following expression for the mean velocity:

$$\bar{v} = (1-\rho)^3 \cdot \left( D + \frac{1-D}{3} \right) + 2 \cdot (1-\rho)^2 \cdot \rho \cdot \left( D + \frac{1-D}{2} \right) + (1-\rho) \cdot \rho^2 .$$

Figure 5 shows the analytical solution and some measured graphs for the mean velocity. It is obvious that the analytical solution becomes worthless for narrow corridors. The reason is the effect of the boundary lanes on the mean velocity. For corridors with an infinite width the analytical solution would fit well.

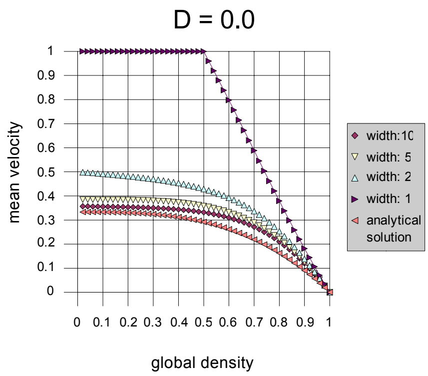


**Figure 5:** Comparison of the analytical solution with measured values for the mean velocity.

## 4 Outlook

An improved analytical solution regarding the boundary lanes is in progress. This will lead to a better fit of the analytical solution to the measured data.

A next step for the simulations will be an investigation of the model regarding obstacles and crossing traffic.

A faster implementation of the model is the basis for finding the lowest jam creating density.

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